**ChE 310 Challenge Problem 1 (10 bonus pts) Due Wed 9/25/19**

Collect all m-files in a single .zip file and upload the .zip file to the course webpage by midnight on Wednesday, September 25, 2019. Please note any collaborations in the Canvas upload comment box. Each student must upload their own individual copy of the work.

One of Dr. Roling’s good friends (and a fellow Iowa State ChE alum) was recently a winner of The Price is Right showcase, taking home a new car and a Caribbean vacation (among other prizes). Before he earned those prizes, however, he had to win the “[Showcase Showdown](https://priceisright.fandom.com/wiki/Showcase_Showdown)” against two other contestants. Let’s take a look at the strategy involved in winning the Showcase Showdown.

*Outline of the problem*. Contestants participating in the Showcase Showdown spin the “Big Wheel” to obtain a score from $0.05 to $1.00 in intervals of $0.05. We will assume for this problem that the odds of spinning any of the 20 possible numbers are exactly the same. (You could think of using the “randi” function to generate numbers for this problem.)

Contestants take one spin of the wheel to obtain their initial score. Based on that score, they then decide whether they want to take a second spin to be added to the result of their first spin. The contestant with the highest score (after one or two spins) is the winner; however, a contestant whose two spins total more than $1.00 automatically loses (you can think of this as resetting their score to exactly $0.00). This creates a strategy for each contestant to maximize their chance of winning by getting as close to $1.00 as possible without going over. There is also a significant advantage to being the last contestant to spin: the first contestant must base their decision to spin again solely on the result of their own first spin. In contrast, the last contestant can compare the result of their first spin with the result of the other player(s) before deciding whether to spin again.

If contestants are tied, they enter a “*spin-off*” in which all tied members have an exactly equal probability of winning. (For this problem, a spin-off may be simulated by choosing a winner at random between any tied contestants.)

For this problem, we do not consider any monetary bonuses associated with spins, but only the objective that players want to maximize their chance of winning this Showcase Showdown.

**(A)** Let’s assume there are only two contestants, Player 1 and Player 2. Player 1 spins the wheel once and obtains an initial score. It is straightforward to show that Player 1 should spin again only if their first spin is less than $0.55; otherwise, Player 1 should hold at the result of their first spin. Player 2 will then spin once and make a decision:

* If Player 2’s first spin is less than Player 1’s total, then Player 2 spins again.
* If Player 2’s first spin is greater than Player 1’s total (or if Player 1 “busted” over $1.00), then Player 2 stops and wins.
* If Player 2’s score is equal to Player 1’s total, then Player 2 spins again IF that total is less than $0.55. Otherwise, Player 2 chooses to accept the tie and enters a spin-off with Player 1.

The winner of this two-person showdown is determined by whichever player has a larger total (without going over $1.00). Assuming players adopt these strategies, **show numerically that Player 1 has a roughly 45.8% chance to win the showdown** (and Player 2 therefore has a 54.2% chance). Ensure you conduct enough trials that you are confident your simulation is accurate to the given precision.

(B) The actual game show has three contestants. The players therefore adopt the following optimal strategy:

* Player 1 spins a second time only if Player 1’s first spin is less than $0.70.
* Player 2 spins a second time if at least one of the following is true:
	+ Player 2’s first spin is less than Player 1’s total
	+ Player 2’s first spin is less than $0.55
	+ Player 2’s first spin is tied with Player 1 at a value less than $0.70.
* Player 3 spins a second time if at least one of the following is true:
	+ Player 3’s first spin is less than Player 1’s total or Player 2’s total
	+ Player 3’s first spin is tied with ONE other player at a value less than $0.55 (otherwise, accept a spin-off)
	+ Player 3’s first spin is tied with BOTH other players at a value less than $0.70 (otherwise, accept a spin-off)

**(B1):** **Show numerically the origins of these optimal strategies**. (Hint: you should find the probability that Player X wins given that their first spin was a particular number, and show how that probability changes depending on the value of the first spin. Assume other players adopt their optimal strategies.)

**(B2):** Based on the optimal strategies shown above, **what percent chance does each player have of winning the Showcase Showdown**? (The result of B1 is not needed to solve B2).

(C) For those really looking to solve a problem, now assume the game adds an additional contestant such that 4 people compete.

**(C1):** **Numerically derive the optimal strategies for these 4 contestants**.

**(C2):** Based on these optimal strategies, **what percent chance does each player have of winning the Showcase Showdown**? (The result of C1 is needed to solve C2).