Lecture 5 In class activity – Laplace Transfers and Transfer Functions

NAMES:_____

Do the following, assuming that q is constant:

a) Put each of the following equations into standard form
b) transform each equation into the Laplace domain
c) define the time constant and the gain(s) in terms of the parameters given
d) find the transfer function between the specified variables.

1. Find $C'(s)/C_i'(s)$ (output / input)

$$\frac{dC'}{dt} = \frac{q}{V}C_i' - \frac{q}{V}C'$$ ← First order ODE

$$sC'(s) = \frac{q}{V}C_i'(s) - \frac{q}{V}C'(s)$$

$$C'(s)\left(s + \frac{q}{V}\right) = \frac{q}{V}C_i'(s) \Rightarrow \frac{C'(s)}{C_i'(s)} = \frac{q/V}{(s + q/V)} = \frac{1}{\left(\frac{V}{q}s + 1\right)}$$

$\frac{K}{\tau s + 1}$ ← gain; time constant

2. Find $C'(s)/C_i'(s)$

$$\frac{dC'}{dt} = \frac{q}{V}C_i' - \left(\frac{q}{V} + 2k_2\bar{C}\right)C'$$

Hint: let $\left(\frac{q}{V} + 2k_2\bar{C}\right) = \frac{1}{\beta}$

$$sC(s) = \frac{q}{V}C_i(s) - \frac{1}{\beta}C'(s)$$

$$C(s)\left(s + 1/\beta\right) = \frac{q}{V}C_i(s) \Rightarrow \frac{C'(s)}{C_i'(s)} = \frac{(q/V)}{(s + 1/\beta)} = \frac{(\beta q/V)}{(\beta s + 1)}$$

gain; time constant

3. Find $T'(s)/T_i'(s)$ and $T'(s)/Q'(s)$

$$\frac{dT'}{dt} = \frac{q}{V}(T_i' - T') + \frac{Q'}{\rho V C_p}$$

Hints: let $\beta = \frac{q}{V}$ and $\alpha = \frac{1}{\rho V C_p}$

$$sT(s) = \beta\left(T_i'(s) - T(s)\right) + \alpha Q'(s)$$

$$T(s)(s + \beta) = \beta T_i'(s) + \alpha Q'(s)$$

$$\left.\frac{T'(s)}{T_i'(s)}\right|_{Q'(s)=0} = \frac{\beta}{(s+\beta)} = \frac{1}{(1/\beta s + 1)}$$

$$\left.\frac{T'(s)}{Q'(s)}\right|_{T_i'=0} = \frac{\alpha}{(s+\beta)} = \frac{(\alpha/\beta)}{(1/\beta s + 1)}$$

$G_1(s)$