

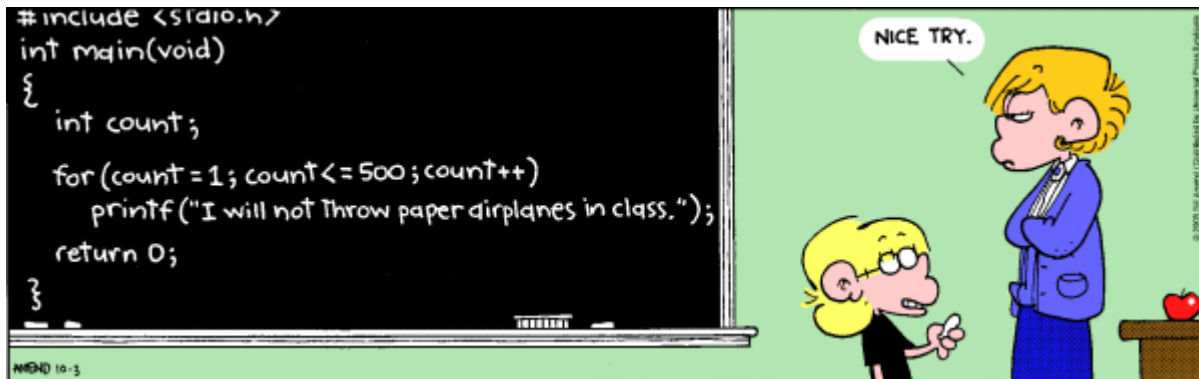
## FINAL EXAM – DO NOT OPEN UNTIL INSTRUCTED

NAME: Key (NFR)

By writing my name, I certify that I have abided by all academic honesty policies.

- This portion of the exam is closed book, closed course notes. No additional resources may be consulted to complete this portion of the exam.
- This portion of the exam is worth 20 points (2 per problem).
- Write your answers to be graded in the space provided.
- You must turn in this portion of the exam before you will receive the free response portion, which is worth an additional 60 points.

Enjoy this comic while you wait:

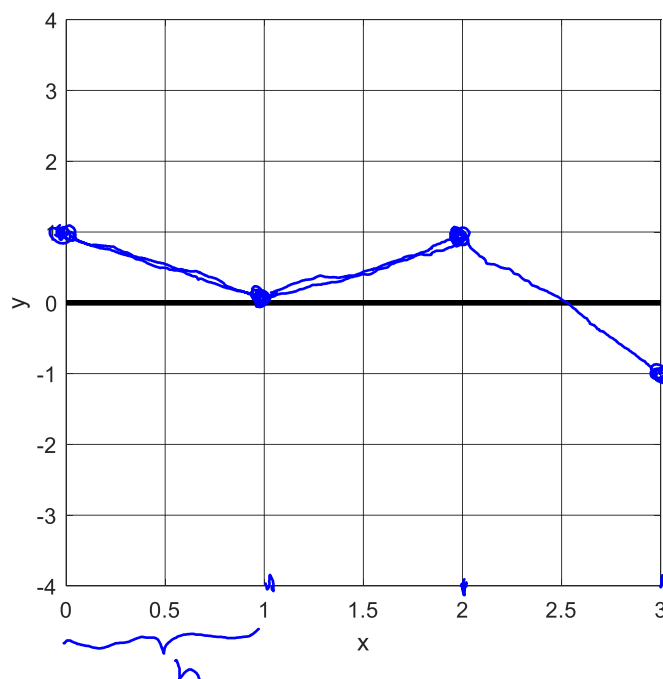


Source: Foxtrot

(1) Consider the following differential equation:

$$\frac{dy}{dx} = -y^2 + x$$

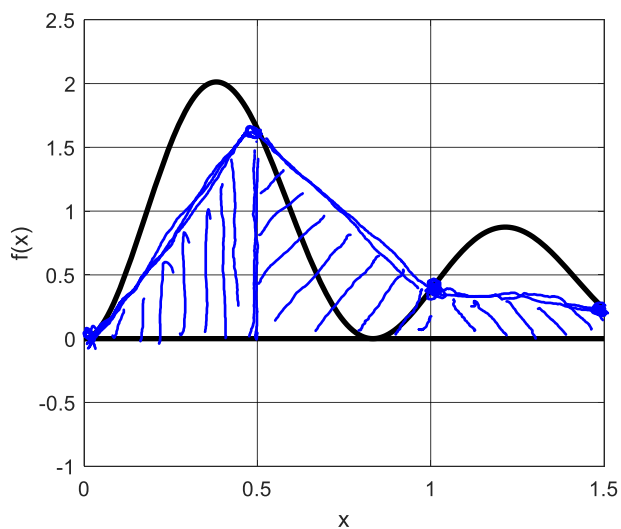
Given that  $y(0) = 1$  and using a step size  $h = 1$ , perform two iterations of Euler's method and show on the axes below.



$x$	Slope
0	$-(1)^2 + 0 = -1$
1	$-(0)^2 + 1 = 1$
2	$-(1)^2 + 2 = 1$
3	$-(1)^2 + 3 = 2$

(2) Consider the following function. Sketch the following integral using the composite trapezoid method, with  $h = 0.5$ .

$$\int_0^{1.5} f(x) dx$$



(3) We are given a large set of evenly spaced, highly accurate and precise measurements of reaction rate as a function of  $T$  and  $P$ . We wish to determine the reaction rate at a value  $T_A$  and  $P_A$  for which we don't have a tabulated rate, although  $T_A$  and  $P_A$  are between values that we have already in our data set.

Should we use interpolation or regression to determine our unknown rate, and why?

Interpolation is used for finding points between values of accurate, tabulated data.

(4) Assume  $f(x)$  is a smooth, continuous, differentiable function of a single variable  $x$ . If  $x_{min}$  represents a value such that  $f$  has a **global minimum** value at  $x = x_{min}$ , which of the following is always true?

(A) MATLAB's `fminsearch` function in open mode will always find  $x_{min}$ .

(B) The function evaluated at  $x_{min}$  is equal to zero; i.e.,  $f(x_{min}) = 0$

(C)  $x_{min}$  is also a root of  $f(x)$

(D) The first derivative of  $f$  evaluated at  $x_{min}$  is equal to zero; i.e.,  $f'(x_{min}) = 0$

(5) Consider the following nonlinear equation; we obtain measurements of  $k$  as a function of  $c$ . Rearrange the expression to a format suitable for linear regression, i.e. the format  $y = a_0 + a_1x$ . What would you plot as your 'x' and 'y'? After regression, how can you obtain  $k_m$  and  $c_s$  from your linear fit?

$$k(c_s + c^2) = k_m c^2 \quad k = \frac{k_m c^2}{c_s + c^2}$$

$$\frac{c_s + c^2}{c^2} = \frac{k_m}{k} \quad \begin{matrix} m & x & b & y \\ \downarrow & \downarrow & \downarrow & \downarrow \end{matrix}$$

$$\frac{c_s}{c^2} + 1 = \frac{k_m}{k} \Rightarrow \frac{c_s}{k_m} \left( \frac{1}{c^2} \right) + \frac{1}{k_m} = \left( \frac{1}{k} \right)$$

You should plot  $\left( \frac{1}{c^2} \right)$  as  $x$  and  $\left( \frac{1}{k} \right)$  as  $y$  for your data. Then use the regressed slope<sup>(m)</sup> and intercept<sup>(b)</sup>:

$$k_m = \frac{1}{b} \quad \text{and} \quad c_s = k_m \cdot m$$

(6) What is the value of  $n$  after running the following code?  $n = \underline{\quad 6 \quad}$

```
n = 0;
for ii = 1:2:5
    for jj = linspace(1,2,2)
        n = n+1;
    end
end
```

$\swarrow [1, 3, 5]$

$\nwarrow [1, 2]$

Because nested  $3 \cdot 2 = 6$

(7) You are in charge of optimizing a new process that is affected by multiple input variables. What are the key advantages (list two) of using design of experiments (DoE) to tackle this problem?

① Efficient resource use - a reduced # of experimental trials needed

② You'll be able to observe interactions between variables.

(8) What does the  $O(h^n)$  notation refer to in a table of numerical derivatives? Why does it matter?

This refers to truncation error. It arises from approximating any function as a Taylor series, and showing which terms are neglected. The 'h' is your step size and refers to how much your error will reduce based on change in step size. A higher 'n' refers to a MORE accurate method.

(9) Using elements of the following matrix 'M', write a Matlab expression that would give you: ans = 6 (there are MANY correct answers for this problem...)

Note to grader.

There are lots of them.

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 4 \\ 8 & 9 & 10 \end{bmatrix}$$

examples

$$M(1,2) + M(1,1)$$

$$M(3,3) - M(2,3)$$

(10) Given an ODE and initial value, and the algorithm options below, in which order would you attempt them? What is the difference between each?

Algorithm	Order	Unique features of this algorithm
Euler method or rk4sys	3	Not efficient method, but harder to diverge, especially if you have a small step size.
ode23s	2	Used to solve STIFF problems.
ode45	1	Robust runge kutia method, typically works and is most efficient.