Tuning Parameter Correlations

1. IMC (Internal Model Correlation) Tuning

Standard Tuning: $\tau_C = \text{Max}(0.1\tau_p, 0.8\theta_p)$ FOPDT

Conservative: $\tau_C = \text{Max}(0.5\tau_p, 4\theta_p)$

	K_C	τ_I	τ_D	α
P	NA			
PI	$\frac{1}{K_p}\left(\frac{\tau_p}{\theta_p + \tau_c}\right)$	τ_p		
PID ideal	$\frac{1}{K_p}\left(\frac{\tau_p + 0.5\theta_p}{\tau_c + 0.5\theta_p}\right)$	$\tau_p + 0.5\theta_p$	$\frac{\tau_p\theta_p}{2\tau_p + \theta_p}$	
PID interacting	$\frac{1}{K_p}\left(\frac{\tau_p}{\tau_c + 0.5\theta_p}\right)$	τ_p	$0.5\theta_p$	
PID ideal w/filter	$\frac{1}{K_p}\left(\frac{\tau_p + 0.5\theta_p}{\tau_c + \theta_p}\right)$	$\tau_p + 0.5\theta_p$	$\frac{\tau_p\theta_p}{2\tau_p + \theta_p}$	$\left(\frac{\tau_c}{\tau_p}\right)\left(\frac{\tau_p + 0.5\theta_p}{\tau_c + \theta_p}\right)$
PID interacting w/filter	$\frac{1}{K_p}\left(\frac{\tau_p}{\theta_p + \tau_c}\right)$	τ_p	$0.5\theta_p$	$\frac{\tau_c}{\tau_c + \theta_p}$

2. ITAE Correlations

: "Integral of the Time Weighted Absolute Error" [FOPDT and parallel form of PID]

$$K_c = \frac{A}{K_p}\left(\frac{\theta_p}{\tau_p}\right)^B \qquad \left\{\frac{1}{\tau_I} = \frac{A}{\tau_p}\left(\frac{\theta_p}{\tau_p}\right)^B\right\}_{\text{Regulatory}} \qquad \left\{\frac{1}{\tau_I} = \frac{1}{\tau_p}\left[A + B\left(\frac{\theta_p}{\tau_p}\right)\right]\right\}_{\text{Servo}} \qquad \tau_D = A\tau_p\left(\frac{\theta_p}{\tau_p}\right)^B$$

Type of Input	Type of Controller	Mode	A	B
Disturbance	P	P	0.490	-1.084
Disturbance	PI	P I	0.859 0.674	-0.977 -0.680
Disturbance	PID	P I D	1.357 0.842 0.381	-0.947 -0.738 0.995
Set point	P	P	0.202	-1.219
Set point	PI	P I	0.586 1.03	-0.916 -0.165
Set point	PID	P I D	0.965 0.796 0.308	-0.95 -0.1465 0.929

PI Controller Tuning Map

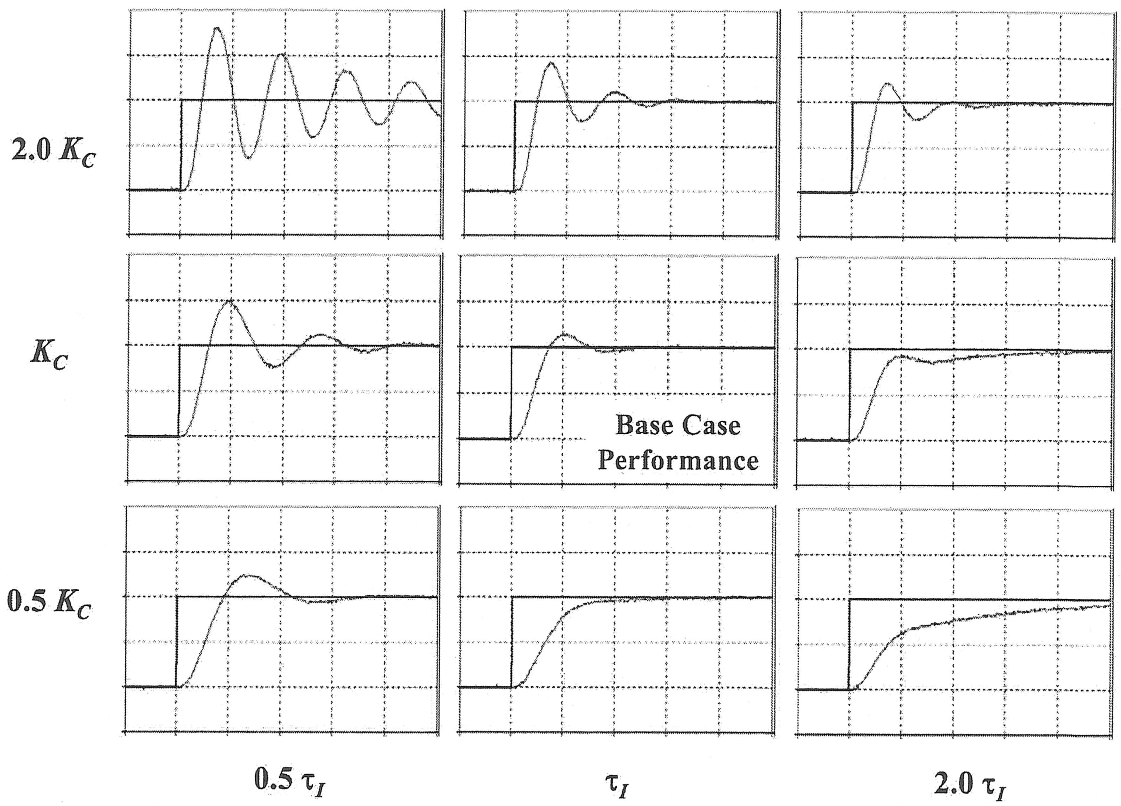


Figure 8.9 – How PI controller tuning parameters impact set point tracking performance

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s+1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2}{\tau_c}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
C	$\frac{K}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s+1)}{\tau^2 s^2+2\zeta\tau s+1},\ \beta>0$	$\frac{2\zeta\tau}{\tau_c+\beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s+1)}$	$\frac{2\tau_c+\tau}{\tau_c^2}$	$2\tau_c+\tau$	$\frac{2\tau_c\tau}{2\tau_c+\tau}$
G	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau}{\tau_c+\theta}$	τ	—
H	$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\tau+\frac{\theta}{2}}{\tau_c+\frac{\theta}{2}}$	$\tau+\frac{\theta}{2}$	$\frac{\tau\theta}{2\tau+\theta}$
I	$\frac{K(\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2-\tau_3}{\tau_c+\theta}$	$\tau_1+\tau_2-\tau_3$	$\frac{\tau_1\tau_2-(\tau_1+\tau_2-\tau_3)\tau_3}{\tau_1+\tau_2-\tau_3}$
J	$\frac{K(\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau-\tau_3}{\tau_c+\theta}$	$2\zeta\tau-\tau_3$	$\frac{\tau^2-(2\zeta\tau-\tau_3)\tau_3}{2\zeta\tau-\tau_3}$
K	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_3+\theta}$	$\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau_1\tau_2}{\tau_1+\tau_2+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
L	$\frac{K(-\tau_3 s+1)e^{-\theta s}}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}{\tau_c+\tau_e+\theta}$	$2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}$	$\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}+\frac{\tau^2}{2\zeta\tau+\frac{\tau_3\theta}{\tau_c+\tau_3+\theta}}$
M	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\theta$	—
N	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c+\theta}{\left(\tau_c+\frac{\theta}{2}\right)^2}$	$2\tau_c+\theta$	$\frac{\tau_c\theta+\frac{\theta^2}{4}}{2\tau_c+\theta}$
O	$\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{2\tau_c+\tau+\theta}{(\tau_c+\theta)^2}$	$2\tau_c+\tau+\theta$	$\frac{(2\tau_c+\theta)\tau}{2\tau_c+\tau+\theta}$

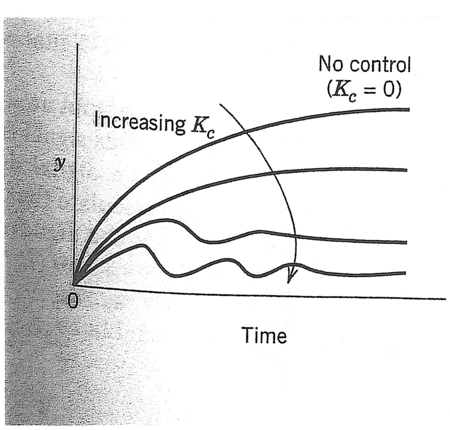


Figure 8.13 Proportional control: effect of controller gain.

Table 8.1 Common PID Controllers

Controller Type	Other Names Used	Controller Equation	Transfer Function
Parallel	Ideal, additive, ISA form	$p(t) = \bar{p} + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)\,dt^* + \tau_D\frac{de(t)}{dt}\right)$	$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$
Parallel with derivative filter	Ideal, realizable, ISA standard	*See Exercise 8.10(a)*	$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s + 1}\right)$
Series (historic)	Multiplicative, interacting	*See Exercise 8.11*	$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)(\tau_D s + 1)$
Series with derivative filter	Physically realizable	*See Exercise 8.10(b)*	$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{\tau_D s + 1}{\alpha\tau_D s + 1}\right)$
Expanded	Noninteracting	$p(t) = \bar{p} + K_c e(t) + K_I\int_0^t e(t^*)\,dt^* + K_D\frac{de(t)}{dt}$	$\frac{P'(s)}{E(s)} = K_c + \frac{K_I}{s} + K_D s$
Parallel, with proportional and derivative weighting	Ideal β, γ controller	$p(t) = \bar{p} + K_c\left(e_P(t) + \frac{1}{\tau_I}\int_0^t e(t^*)\,dt^* + \tau_D\frac{de_D(t)}{dt}\right)$ where $e_P(t) = \beta y_{sp}(t) - y_m(t)$, $e(t) = y_{sp}(t) - y_m(t)$, $e_D(t) = \gamma y_{sp}(t) - y_m(t)$	$P'(s) = K_c\left(E_P(s) + \frac{1}{\tau_I s}E(s) + \tau_D s E_D(s)\right)$ where $E_P(s) = \beta Y_{sp}(s) - Y_m(s)$, $E(s) = Y_{sp}(s) - Y_m(s)$, $E_D(s) = \gamma Y_{sp}(s) - Y_m(s)$

↑ β and γ typically range 0 to 1

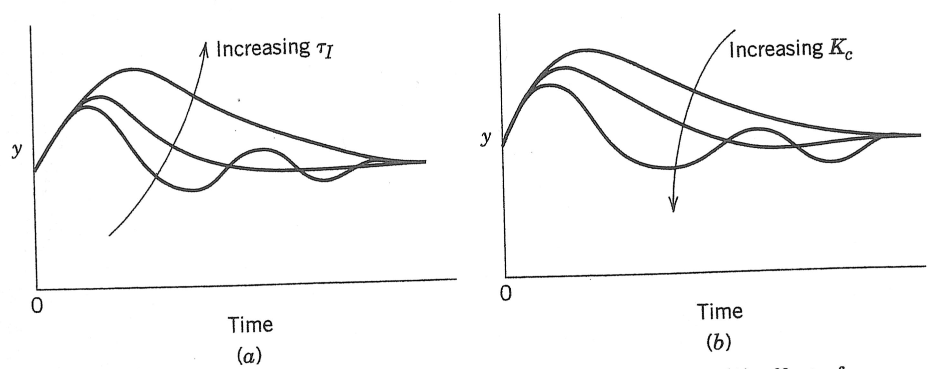
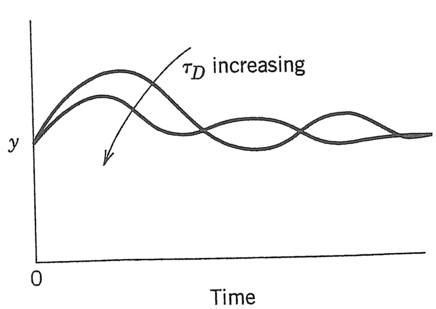


Figure 8.14 Proportional-integral control: (*a*) effect of integral time, (*b*) effect of controller gain.



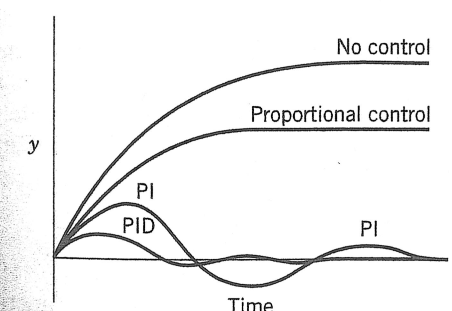
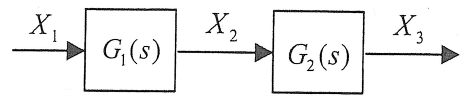


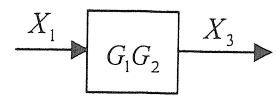
Figure 8.15 PID control: effect of derivative time.

Table 1. Block Diagram Transformations [Taken from Dorf & Bishop Textbook]

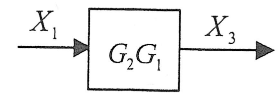
Transformation	Original Diagram	Equivalent Diagram

1. Combining blocks in cascade

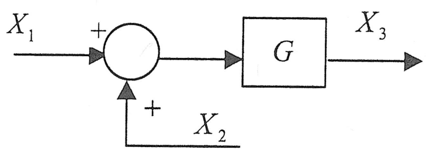


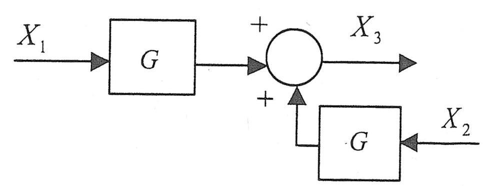


or

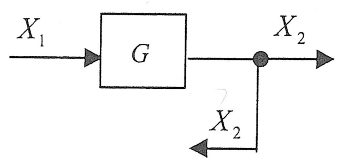


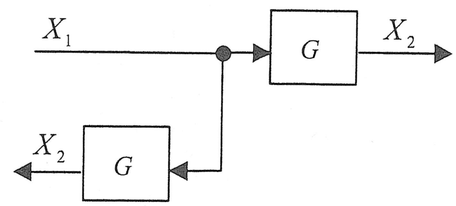
2. Moving a summing point behind a block



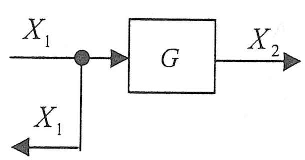


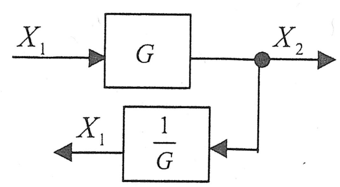
3. Moving a pickoff point ahead of a block



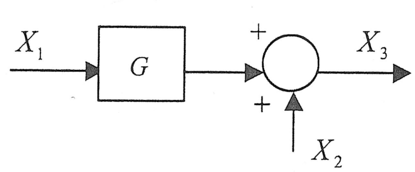


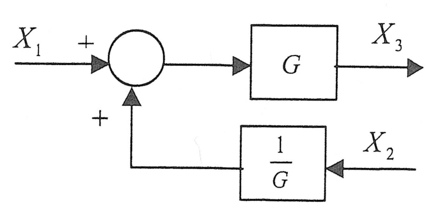
4. Moving a pickoff point behind a block





5. Moving a summing point ahead of a block





6. Eliminating a feedback loop