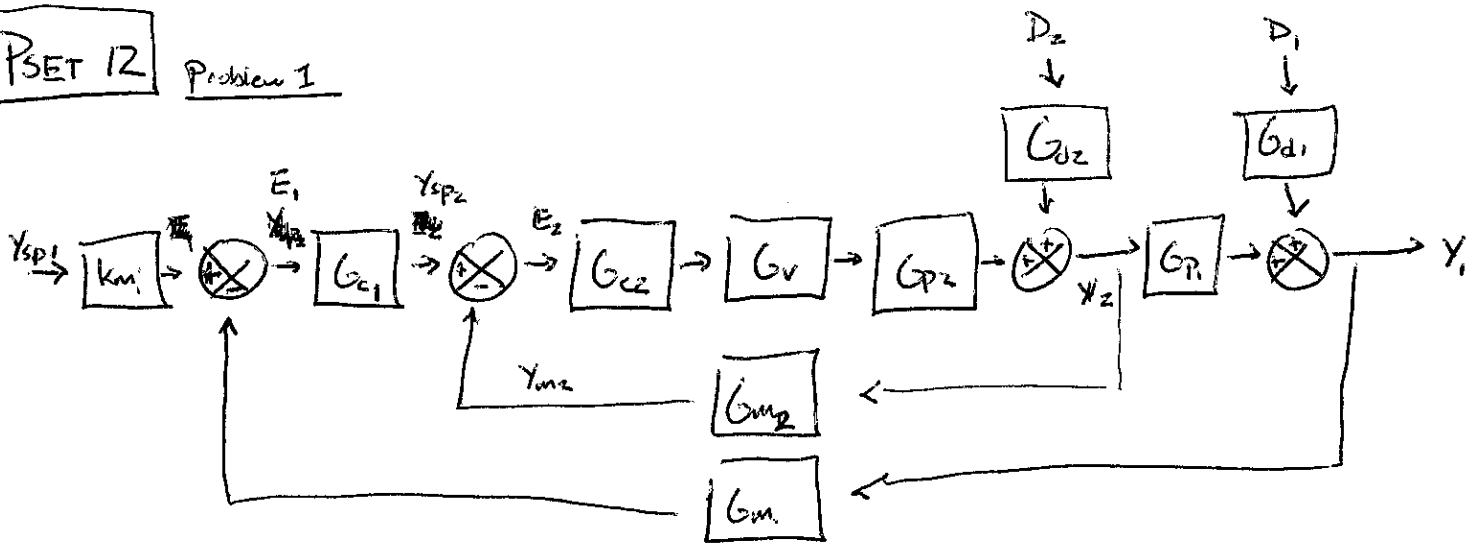
# PSET 12 Problem 1




inner loop

Assume $Y_{sp_1} = 0$ and $D_1 = 0$

$$Y_1 = G_{p_1} Y_2$$

$$Y_2 = G_{D_2} D_2 + G_{p_2} G_v G_{c_2} E_2$$

$$E_1 = -G_m Y_1 \qquad E_2 = G_{c_1} E_1 - G_{m_2} Y_2$$

eliminate all variables except $Y_1$ and $D_2$:

$$Y_2 = G_{D_2} D_2 + G_{p_2} G_v G_{c_2} (G_{c_1} E_1 - G_{m_2} Y_2)$$

$$Y_2 = \frac{G_{D_2} D_2 + G_{p_2} G_v G_{c_2} G_{c_1} E_1}{1 + G_{p_2} G_v G_{c_2} G_{m_2}} = \frac{G_{D_2} D_2 - G_{p_2} G_v G_{c_2} G_{c_1} G_{m_1} Y_1}{1 + G_{p_2} G_v G_{c_2} G_{m_2}}$$

$$Y_1 = \frac{G_{p_1} G_{D_2} D_2 - G_{p_1} G_{p_2} G_v G_{c_2} G_{c_1} G_{m_1} Y_1}{1 + G_{p_2} G_v G_{c_2} G_{m_2}}$$

$$\frac{Y_1}{D_2} = \frac{G_{p_1} G_{D_2}}{1 + G_{p_2} G_v G_{c_2} G_{m_2} + G_{p_1} G_{p_2} G_v G_{c_2} G_{c_1} G_{m_1}}$$

↙ inner loop disturbance

$$\frac{Y_2}{Y_{sp_2}} = \frac{G_{c_2} G_v G_{p_2}}{1 + G_{c_2} G_v G_{p_2} G_{m_2}}$$

← S.P. change for inner loop (can do this one by inspecting the diagram

Now, let's do the outer loop:

$$\frac{Y_1}{Y_{sp_1}} = \frac{G_{c_1} G_{inner} G_{p_1}}{1 + G_{c_1} G_{inner} G_{p_1} G_{m_1}} = \frac{\dfrac{G_{c_1} G_{c_2} G_v G_{p_1} G_{p_2}}{1 + G_{c_2} G_v G_{p_2} G_{m_2}}}{1 + \dfrac{G_{c_1} G_{c_2} G_v G_{p_1} G_{p_2} G_{m_1}}{1 + G_{c_2} G_v G_{p_2} G_{m_2}}}$$

$$\frac{Y_1}{Y_{sp_1}} = \frac{G_{c_1} G_{c_2} G_v G_{p_1} G_{p_2}}{1 + G_{c_2} G_v G_{p_2} G_{m_2} + G_{c_1} G_{c_2} G_v G_{p_1} G_{p_2} G_{m_1}}$$

Transfer function for cascade S.P. change

Likewise for disturbance

$$\frac{Y_1}{D_1} = \frac{G_{D_1}(1 + G_{c_2} G_v G_{p_2} G_{m_2})}{1 + G_{c_2} G_v G_{p_2} G_{m_2} + G_{c_1} G_{c_2} G_v G_{p_2} G_{p_1} G_{m_1}}$$

# Group Problem 1

10.6

$R = e^{-\mu t}$

$[P = 1 - R]$

| | failure rate | Reliability (R) | P (Prob. of failure) |
|---|---|---|---|
| Solenoid | 0.01 | 0.99 | 0.01 |
| Level switch | 0.45 | 0.64 | 0.36 |
| level alarm | 0.3 | 0.74 | 0.26 |

~~$R_{sys} = (0.99)(0.64)(0.74) = 0.47$~~ No! They are not in series. The solenoid and level switch are in series.

$R_{switch\,sys} = (0.99)(0.64) = 0.63$

$P_{switch\,sys} = 0.37$

The level alarm is independent (or in parallel)

$P_{sys} = (0.37)(0.26) = \boxed{0.096}$ ← Probability of failure of this system

MTBF

$R_{sys} = 1 - P_{sys} = 0.904$

$\mu = -\ln(R)$

$\mu = 0.1$ failures per year

$= \frac{1}{\mu} = 10$ years between failures of the pump due to cavitation (on average)

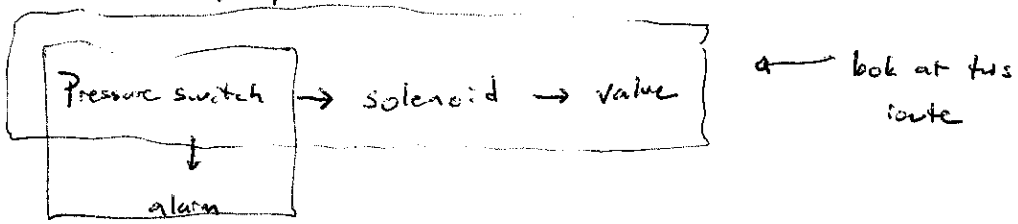
# Group Problem 2

**10.4** Reliability and MTBF

SS = $\mu = 0.42$

V $\mu = 0.42$

Failure of high pressure interlock system ...




← look at this route

ignore this route as the alarm failure rate is low and if this piece failed, you would still be ok in terms of an interlock

$R = \exp(-\mu t)$

| | Failure Rate ($\mu$) | Reliability (R) |
|---|---|---|
| switch | 0.34 | 0.65 |
| Solenoid | 0.42 | 0.65 |
| valve | 0.42 | 0.66 |

$R_{sys} = (0.65)(0.65)(0.66) = 0.28$ ← Reliability

↑ in years

Probability of failure = 1 − R = 72 %

$\mu_{sys} = -\ln(R) = 1.27$ failures/yr

$MTBF = \frac{1}{\mu_{sys}} = \frac{1}{1.27} = 0.79$ yr or 287 days between failures.