PSET 4 – ChE 421 – Due 9.18.2018 at 12:45PM

NOTE: there will be no class on 9.18 due to Career Fair; submit your individual problem set **electronically** as a **single PDF** **with the following naming structure LastName_FirstName_PSET4** as an attachment to the following email address:

ChE_421.d8j8ou53k9qbn80n@u.box.com

The group problems should be sent to Dillon via SLACK.

1. Solve:

A heater for a semiconductor wafer has first-order dynamics, that is, the transfer function relating changes in temperature T to changes in the heater input power level P is

$$\frac{T'(s)}{P'(s)} = \frac{K}{\tau s + 1}$$

where K has units [°C/Kw] and τ has units [min].

The process is at steady state when an engineer changes the power input stepwise from 1 to 2 kW. She notes the following:

(i) The process temperature initially is 100 °C.

(ii) Four minutes after changing the power input, the temperature is 400 °C.

(iii) Thirty minutes later the temperature is 500 °C.

(a) What are K and τ in the process transfer function?

(b) If at another time the engineer changes the power input linearly at a rate of 0.5 kW/min, what can you say about the maximum rate of change of process temperature: When will it occur? How large will it be?

2. Solve:

The dynamic behavior of the liquid level in a leg of a manometer tube, responding to a change in pressure, is given by

$$\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t)$$

where $h'(t)$ is the level of fluid measured with respect to the initial steady-state value, $p'(t)$ is the pressure change, and R, L, g, ρ, and μ are constants.

(a) Rearrange this equation into standard gain-time constant form and find expressions for K, τ, ζ in terms of the physical constants.

(b) For what values of the physical constants does the manometer response oscillate?

(c) Would changing the manometer fluid so that ρ (density) is larger make its response more oscillatory, or less? Repeat the analysis for an increase in μ (viscosity).

3. Solve

A step change from 15 to 31 psi in actual pressure results in the measured response from a pressure-indicating element shown in Fig. E5.14.

(a) Assuming second-order dynamics, calculate all important parameters and write an approximate transfer function in the form

$$\frac{R'(s)}{P'(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

where R' is the instrument output deviation (mm), P' is the actual pressure deviation (psi).

(b) Write an equivalent differential equation model in terms of actual (not deviation) variables.

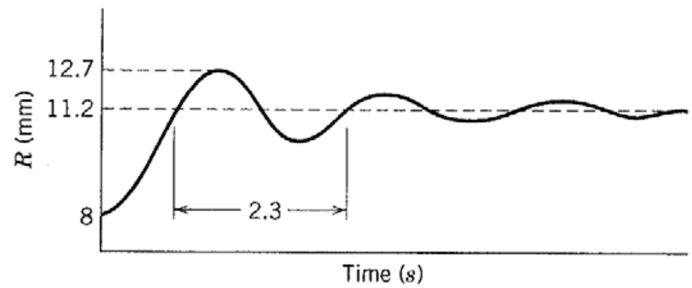


Figure E5.14

4. Solve

Consider the transfer function:

$$G(s) = \frac{0.7(s^2 + 2s + 2)}{s^5 + 5s^4 + 2s^3 + 4s^2 + 6}$$

(a) Plot its poles and zeros in the complex plane. A computer program that calculates the roots of the polynomial (such as the command *roots* in MATLAB) can help you factor the denominator polynomial.

(b) From the pole locations in the complex plane, what can be concluded about the output modes for any input change?

(c) Plot the response of the output to a unit step input. Does the form of your response agree with your analysis for part (b)? Explain.

5. Solve

A process consists of an integrating element operating in parallel with a first-order element (Fig. E6.6).

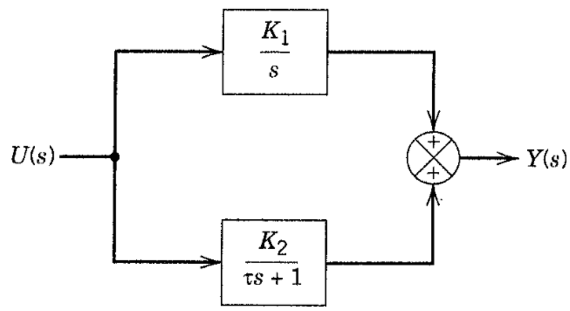


Figure E6.6

(a) What is the order of the overall transfer function, $G(s) = Y(s)/U(s)$?

(b) What is the gain of $G(s)$?

(c) What are the poles of $G(s)$? Where are they located in the complex s-plane?

(d) What are the zeros of $G(s)$? Where are they located? Under what condition(s) will one or more of the zeros be located in the right-half s-plane?

(e) For what conditions, will this process exhibit both a negative gain and a right-half plane zero?

(f) For any input change, what functions of time (response modes) will be included in the response, $y(t)$?

(g) Is the output bounded for any bounded input change, for example, $u(t) = M$?

6. Solve

For the process described by the exact transfer function

$$G(s) = \frac{5}{(10s + 1)(4s + 1)(s + 1)(0.2s + 1)}$$

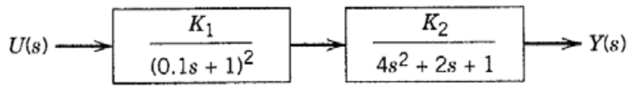
(a) Find an approximate first-order-plus-time-delay transfer function model.

(b) Simulate and plot the response $y(t)$ of both the approximate model and the exact model on the same graph for a unit step change in input $x(t)$.

(c) What is the maximum error between the two responses? When does it occur?

7. Solve

A process has the following block diagram representation:



(a) Will the process exhibit overshoot for a step change in u? Explain/demonstrate why or why not.

(b) What will be the approximate maximum value of y for $K = K_1K_2 = 1$ and a step change, $U(s) = 3/s$?

(c) Approximately when will the maximum value occur?

(d) Simulate and plot both the actual fourth-order step response and the response for a second-order-plus-time delay model that approximates the fourth order system. Using the same second-order-plus-time delay model, what happens when the time constant in the first block is changed to 1 and then 5? Compare the step responses graphically.

8. GROUP Problem – work with your team submit 1 solution to Dillon by deadline via SLACK

A vertical, cylindrical tank is filled with water at 20 °C. The tank is insulated at the top and bottom, with diameter of 0.5 m and height of 1.0 m. The overall heat transfer coefficient is $U = 120$ W/m^{2}K. The density of water is $\rho = 1000$ kg/m^3, the heat capacity $C_p = 4180$ J/kgK, the melting point is 0 °C and the heat of fusion is $\lambda = 334$ kJ/kg.

(a) In the evening, the tank is suddenly exposed to air at −15 °C. Calculate how many minutes it will take for the first crystal of ice to form in the tank. Model this process by assuming thermal equilibrium between the tank and the environment at an initial steady state, followed by a sudden drop in the outside temperature to −15 °C.

(b) How long will it take to completely freeze the water in the tank? You many neglect any volume expansion associated with freezing and assume that the tank is well-mixed, that is, the temperature is uniform within the tank and there are no radial temperature gradients.

9. GROUP Problem – work with your team submit 1 solution to Dillon by deadline via SLACK

The dynamic behavior of a packed-bed reactor can be approximated by a transfer function model

$$\frac{T'(s)}{T_i'(s)} = \frac{3(2-s)}{(10s+1)(5s+1)}$$

where T_i is the inlet temperature, T is the outlet temperature (°C), and the time constants are in hours. The inlet temperature varies in a cyclic fashion due to the changes in ambient temperature from day to night.

As an approximation, assume that T_i' varies sinusoidally with a period of 24 hours and amplitude of 12 °C. What is the maximum variation in the outlet temperature, T?