The following ODE has been found to govern the behavior of your system output (y) as a function of location (x):

$$\frac{d^{2}y}{dx^{2}}+ \frac{dy}{dx}=x^{2}+2x $$

Where y=1 at x = 0 and y=41.673 at x = 5.

1. Use the space below to derive the finite difference form of this ODE using O(h2) error equations and solve the problem using a spacing of h = 1 (there will be four internal, unknown nodes). Use Matlab to solve your system of equations and plot the y profile from x = 0 to x = 5 using ‘o’ marks to denote the finite difference solutions.
2. Use a numerical ODE solver to solve this same problem. Plot the numerical ODE solver output on same plot as part (a) using ‘+’ marks to denote the ODE solver points.
3. Plot the analytical solution as a solid line on same plot as part (a) and (b). Label axes and insert legend in the plot. Analytical solution is as follows:

$$y=e^{-x}+\frac{x^{3}}{3}$$

1. At what x value does y = 15? NOTE: there are at least three different ways to tackle this question given the work you’ve done in parts a-c. Pick any that is fastest for you.

**function Final\_p2**

**% Coded by NFR on 12.12.17 % Mfile attempt - 2 pts**

**% This code solves the ODE problem on the final**

**% PART A: Finite Difference % Correct derivation - 7 pts**

**% See paper for finite difference work**

**A = [-2 1.5 0 0; 0.5 -2 1.5 0; 0 0.5 -2 1.5; 0 0 0.5 -2]; % Correct A matrix = 2 pts.**

**b = [2.5; 8; 15; -38.5]; % Correct b vector = 1 pt.**

**yin = A\b; % Solution of internal points = 2 pts**

**y = [1; yin; 41.673];**

**x = [0:5]';**

**plot(x,y,'o') % 2 pts = Plot of ALL points, not just the internal unknown pts**

**hold on**

**% PART B**

**% ODE solver, BVP**

**go = 1; % 1 pt - initial guess**

**greal = fzero(@odeMIN,go); % 2 pt - call of minimization function to get best fit slope for BVP conditions**

**tspan = [0 5];**

**y0 = [1 greal];**

**[tp, yp] = ode45(@SYS,tspan,y0); % 2 pt - run ODE solver again with best fit slope**

**plot(tp,yp(:,1),'+') % 2 pt - plot**

**% PART C: Analytical solution**

**F = @(x) exp(-x)+x^3/3; % 1 pt - Data input**

**fplot(F,[0 5]) % 2 pt - plot function**

**xlabel('X')**

**ylabel('Y')**

**legend('Finite Difference','ODE45','Analytical') % 2 pt - label axes**

**hold off**

**% Part D:**

**% At what X position does Y = 15? You can solve this with the analytical**

**% function (root finding), by using the plot, or interpolation with the**

**% ODE45 solution...I will do the latter**

**disp('X value, where Y = 15:') % 3 pt - correct method identified**

**Xval = interp1(yp(:,1),tp,15,'spline') % 2 pt - correct answer**

**end**

**function E = odeMIN(g) % 3 pts - correct setup of minimizing function for BVP**

**tspan = [0 5];**

**y0 = [1 g];**

**[tp, yp] = ode45(@SYS,tspan,y0);**

**E = 41.673-yp(end,1); % We want to find the guess value that minimizes this error, that is matches our end condition**

**end**

**function dy = SYS(t,y) % 4 pts - correct setup of system of ODE for BVP, including change of variable**

**% t = x, y(1) = y, and y(2) = z**

**dy = [y(2);...**

 **t^2+2\*t-y(2)];**

**end**